Autonomous Satellite Navigation at Five Times Synchronous Altitude

Roger A. Metzler*
The Aerospace Corporation, El Segundo, California

The performance of a navigation system operating autonomously onboard a satellite in an orbit of five times (5×) synchronous altitude is simulated and studied. The system utilizes a star sensor and an Earth sensor, which measure star and Earth positions and the Earth subtended angle, to augment the satellite position and attitude estimates maintained by the navigation filter. The technique employed to evaluate the navigation filter performance is a covariance analysis of the optimal Kalman filter. Insight is provided into the qualitative and quantitative characteristics of the autonomous system operating at the 5× altitude. The behavior of the 15 filter state variables is analyzed. Also, analysis of the filter's sensitivity to initial estimation errors and measurement errors reveals the important variable interactions and measurement quantities.

Nomenclature

b	= Earth sensor measurement bias error
B	= standard deviation of b
$\vec{e}$	= vector from satellite to center of Earth
$E[\ ]$	= expected value
ECI	= Earth centered inertial coordinate system
ES	= Earth sensor
H	= measurement matrix for filter model
I	= identity matrix
K	= Kalman gain matrix
N	= standard deviation of Earth sensor measurement random error
P	= filter model estimation error covariance
Q	= filter model process noise covariance
r_E	= radius of Earth
R	= filter model measurement noise covariance
R	= range from satellite to center of Earth
$\hat{s}$	= unit vector to star
SS	= star sensor
TB2S	= body to star sensor coordinate transformation
T12B	= ECI to body coordinate transformation
T12ES	= ECI to Earth sensor coordinate transformation
T12ES	= ECI to Earth sensor coordinate transformation
T12S	= ECI to star sensor coordinate transformation
v	= filter model measurement noise
w	= filter model process noise
x	= filter model state vector
X	= x component of satellite position in ECI coordinates
$\vec{X}$	= satellite position in ECI coordinates
Y	= y component of satellite position in ECI coordinates
z	= filter model measurement vector
Z	= z component of satellite position in ECI coordinates
α	= Earth sensor measurement angle
β	= bearing angle star sensor measurement
Γ	= filter model process noise dynamics matrix
θ	= Earth sensor equivalent measurement angle
λ	= elevation angle star sensor measurement
σ	= standard deviation
ϕ	= satellite attitude or sensor alignment angle
Φ	= filter model state transition matrix

Superscripts and Subscripts

$(\ )^*$	= filter quantity identical to truth or "real world" quantity
$(\ )^-$	= filter quantity before measurement update
$(\ )^+$	= filter quantity after measurement update
$(\ )^T$	= matrix transpose
$(\ )^{-1}$	= matrix inverse
$(\hat{\ })$	= filter estimate, or unit vector
$(\ )_{ECI}$	= quantity expressed in Earth centered inertial coordinates
$(\ )_{ES}$	= quantity expressed in Earth sensor coordinates
$(\ )_k$	= filter quantity at discrete time step k
$(\ )_0$	= initial value
$(\ )_{SS}$	= quantity expressed in star sensor coordinates

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*Member of the Technical Staff, Guidance Analysis Deaprtment, Guidance and Control Division. Member AIAA.

Introduction

An autonomous satellite navigation system is self-contained, operates in real time, is nonradiating, and does not rely on information from ground stations. This study investigates the ability of an autonomous satellite navigation system to estimate the position, velocity, and attitude of a satellite operating in a circular orbit of five times (5×) synchronous altitude.

Previous studies of autonomous navigation have been primarily restricted to low-altitude orbits. The most popular methods attempted were known and unknown landmark tracking.[1-4] Several studies of autonomous navigation at higher altitudes have also been conducted.[5,6] The most comprehensive of these is the High-Altitude Navigation Study (HANS)[6] conducted by Le May et al. It is shown that, at high altitudes, landmark tracking loses its effectiveness because of the small size of the Earth's image and accompanying resolution requirements on the instruments. However, two viable measurement schemes, each utilizing a star sensor and an Earth sensor, are identified for satellite navigation in a 60,000-n.mi. orbit. In one configuration the sensors measure the angle, at the satellite, subtended by the lines of sight to a star and the Earth's horizon. In the second configuration the positions of a star and the Earth relative to the satellite are measured, as is the angle, at the satellite, subtended by the Earth's diameter.

This study extends the work of Ref. 6, examining the characteristics and accuracy of this latter measurement configuation utilized in an autonomous navigation system for a satellite in a circular orbit of 5× synchronous altitude—approximately 114,000 n.mi. The star sensor provides the filter with two measurements—the bearing and elevation angles to a star. The Earth sensor provides three measurements—the two angles which a vector to the center of the Earth makes with the sensor coordinate system, and the distance to the Earth's center, which is inversely proportional to the angle, at the satellite, subtended by the Earth's diameter. These measurements are combined with a priori data to enable the onboard navigation filter to estimate satellite position, velocity, and attitude, as well as several sensor biases. No inertial measurement unit is utilized.

The navigation filter performance is studied using an optimal covariance analysis, which assumes that the filter contains an exact model of the actual or "real world" process dynamics. This optimal analysis yields the minimum error statistics possible for the system configuration assumed. The navigation characteristics studied include general filter behavior, sensitivity to error sources, and the critical filter states and measurements.

Methodology and Models

Optimal Covariance Analysis

The technique employed to evaluate the performacne of the navigation filter is a covariance analysis of the optimal Kalman filter. This technique assumes that the onboard filter models the "real-world" satellite dynamics and measurements exactly; no approximations are made. The filter model is[7]

$$x_{k+1} = \Phi_k^* x_k + \Gamma_k^* w_k^*$$
$$z_k = H_k^* x_k + v_k^* \tag{1}$$

where Φ_k^* and H_k^* are the true state transition and measurement matrices; and w^* and v^* are the uncorrelated, discrete, random process, and measurement noises, with statistics $E(w_k^*) = E(v_k^*) = 0$, $E(w_k^* w_k^{*T}) = Q_k^*$, $E(v_k^* v_k^{*T}) = R_k^*$. If the initial estimation error covariance matrix is

$$P_0^* = E[(\hat{x}_0 - x_0)(\hat{x}_0 - x_0)^T] \tag{2}$$

the error covariance for the linear, unbiased, minimum variance (Kalman) estimate can be propagated between measurements

$$P_{k+1}^{*-} = \Phi_k^* P_k^{*+} \Phi_k^{*T} + \Gamma_k^* Q_k^* \Gamma_k^{*T} \tag{3}$$

and across measurements

$$P_k^{*+} = (I - K_k^* H_k^*) P_k^{*-} \tag{4}$$

where the Kalman gain K_k^* is

$$K_k^* = P_k^{*+} H_k^{*T} R_k^{*-1}$$
$$= P_k^{*-} H_k^{*T} (H_k^* P_k^{*-} H_k^{*T} + R_k^* \times^{-1} \tag{5}$$

The standard deviation (1σ) of the estimation errors are the square roots of the diagonal elements of P^*. Since the filter utilizes an exact model of the physical system, the analysis yields the minimum error statistics which would result from the navigation system configuration in the presence of state variable and measurement errors with the statistics assumed. Any filter modeling errors would derease system accuracy.

The navigation system analyzed here, using a star sensor and an Earth sensor for measurements, consist of a 15 state filter (three ECI position components, three ECI velocity components, three alignment angles, three star sensor alginment biases, an Earth sensor range bias, and two Earth sensor alginment biases); two star sensor measurement angles; and three Earth sensor measurements.

A computer program propagates the error covariance equations. The program inputs include the measurement interval, and the standard deviations of 1) the initial estimation error for each state variable; 2) the noise in each state variable propagation equation (zero in all cases for this study); and 3) the noise in each measurement. The program also requires a nominal trajectory for the satellite. The nearly circular, 5× synchronous altitude, polar orbit studied has the following characteristics: semimajor axis $a = 113{,}839$ n.mi. (691,687,005 ft); eccentricity $e = 0.003146$; inclination $i = 90.005$ deg; right ascension of the ascending node $\Omega = 27.886$ deg; argument of perigee $\omega = 92.410$ deg; time of last perigee $\tau = -233{,}430.99$ s; period $T = 16{,}056.3$ min (11.15 days).

The local circular velocity of the orbit is 4508 ft/s. The vehicle is maintained at a constant inertial attitude.

Star Sensor Model

The star sensor modeled measures the bearing and elevation angles, in the sensor coordinate system, to a star, as shown in Fig. 1. The measurement equations are

$$\beta = \tan^{-1} \frac{s_y}{s_x} + \nu_\beta$$
$$\lambda = \tan^{-1} \frac{s_z}{\sqrt{s_x^2 + s_y^2}} + \nu_\lambda \tag{6}$$

where β and λ are the bearing and elevation angles, and $\hat{s}$ is the unit vector to the star, expressed in star sensor coordinates. The quantities ν_β and ν_λ represents the random instrument noise inherent in each measurement, and are assumed to be zero mean with standard deviations σ_β and σ_λ. The star unit vector in star sensor coordinates is related to the unit vector in ECI coordinates through the transformation:

$$\hat{s}_{SS} = [T12S]\hat{s}_{ECI} \tag{7}$$

where T12S is the transforamtion from ECI to star sensor coordinates, which is maintained by the navigation filter; and $\hat{s}_{ECI}$ is the star unit vector in ECI coordiantes, which is preloaded into the computer prior to flight. (To assure that at least one star is visible to the vehicle at all times, several star unit vectors are preloaded, and a star selected for each measurement based on a minimum geometrical error criterion.)

The inertial to star sensor transformation is the product of two other transforamtions:

$$[T12S] = [TB2S][T12B] \tag{8}$$

where T12B and TB2S are the transformation matrices from ECI to body, and from body to star sensor coordinates,

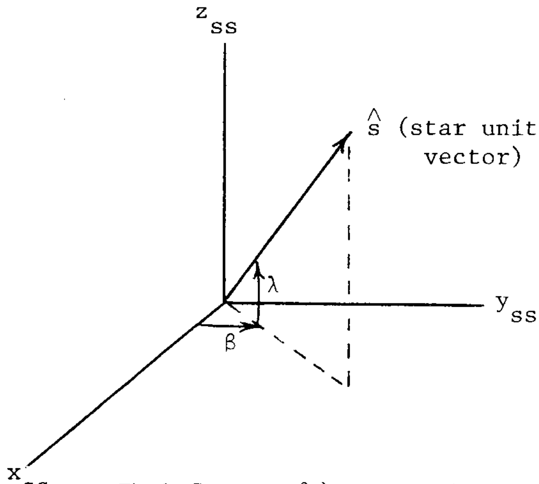

Fig. 1 Star sensor β, λ measurements.

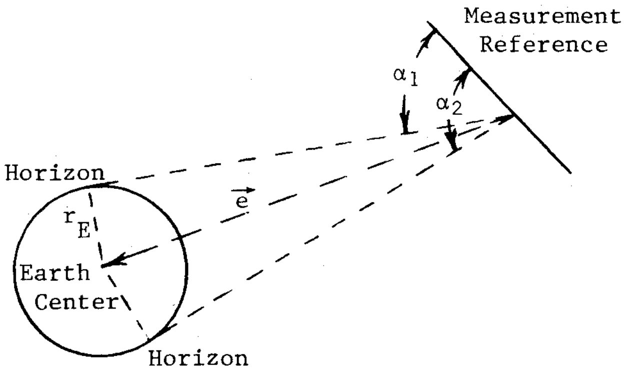

Fig. 2 Earth sensor (four-head) measurement in the orbit plane.

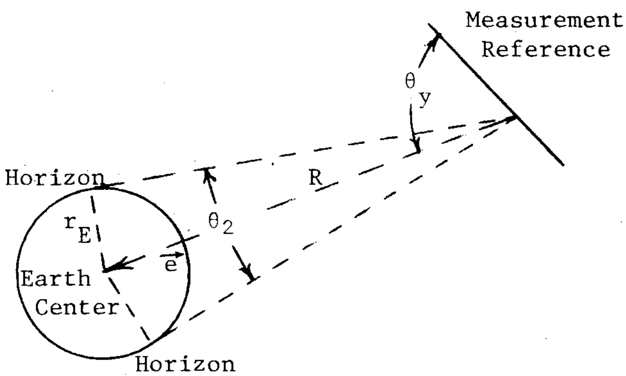

Fig. 3 Earth sensor equivalent θ measurements in the orbit plane.

respectively. If the filter's estimate of either matrix is in error compared to truth, the filter's measurement predictions are in error, contributing to an error in the filter's state estimate. An error in the filter's estimate of the alginment bias of the star sensor relative to the body contributes an error to TB2S. An error in the filter's estimate of the vehicle's attitude relative to the inertial results in an error in T12B.

Earth Sensor Model

The Earth sensor modeled is a four-head horizon sensor which measures the angles between an onboard reference direction and the two Earth horizons. The measurements α_1 and α_2, in the orbital plane, are shown in Fig. 2, and similar measurements, α_3 and α_4 are made normal to the orbit plane. From these measurements, an equivalent set of measurements can be derived using the transformation:

$$\theta_y = (\alpha_1 + \alpha_2)/2$$
$$R = \frac{r_E}{\sin(\theta_2/2)} = \frac{r_E}{\sin[(\alpha_2 - \alpha_1)/2} \simeq \frac{2r_E}{\alpha_2 - \alpha_1} \quad \theta_x = (\alpha_3 + \alpha_4)/2 \tag{9}$$

The measurement θ_y and θ_x are the in-plane and out-of-plane angles from an arbitrary vehicle measurement reference to the vector to the center of the Earth (nadir), and R is the range to the center of the Earth, which is inversely proportional to the Earth's subtended angle at the satellite, θ_2. (At 5× altitude the Earth's subtended angle is approximately 4 deg, so $\sin\theta_2 \simeq \theta_2$ is valid.) The in-plane geometry is shown in Fig. 3.

If the Earth sensor measurement reference is assumed to be roughly aligned with a local horizontal coordinate frame, as shown in Fig. 4, the measurements relate to the components of the vector to the center of the Earth, $\boldsymbol{e}$, as:

$$\theta_x = \cos^{-1} \frac{e_y}{\sqrt{e_y^2 + e_z^2}} + b_{\theta_x} + \nu_{\theta_x}$$
$$R = \sqrt{e_x^2 + e_v^2 + e_z^2} + b_R + \nu_R \tag{10}$$

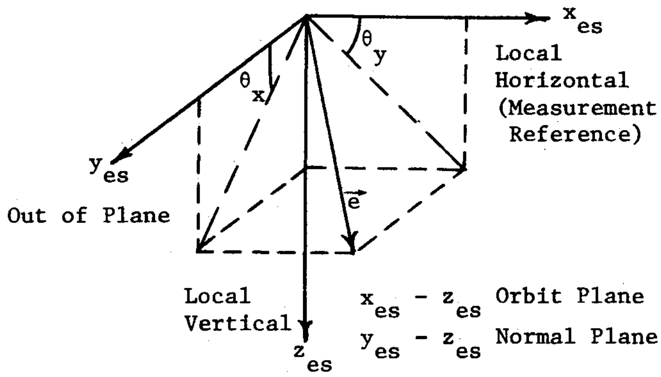

Fig. 4 Earth sensor measurement with a local horizontal measurement reference.

where e_x, e_y, e_z are the Earth center vctor components in Earth sensor coordinates, and b and ν denote the bias and random measurement errors, respectively. Since the navigation filter state vector contains the vehicle's ECI coordinate location, $\vec{X}_{ECI} = (X\ Y\ Z)^T$, and the Earth center vector is opposite the vehicle vector ($\vec{e}_{ECI} = -\vec{X}_{ECI}$), the filter calculates the Earth center vector in Earth sensor coordinates using:

$$\vec{e}_{ES} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = [\text{T12ES}]\vec{e}_{ECI} = -[\text{T12ES}] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ECI} \tag{11}$$

where T12ES is the transformation from ECI (filter) coordinates to Earth sensor coordinates, maintained by the filter.

Using the transformation of Eq. (9), the statistics of the bias and random measurement errors for a four-head Earth sensor model can be transformed into measurement error statistics for the equivalent θ_x, R, θ_y measurement set. If the biases of the four horizon sensor measurements (α) are assumed to be zero mean, uncorrelated, with equal standard deviations, B, then the transformed measurement biases are also zero mean and uncorrelated, with standard deviations:

$$\sigma_{b_{\theta_x}} = \sigma_{b_{\theta_y}} = B/\sqrt{2} \qquad \sigma_{b_R} = K\sqrt{2}B \tag{12}$$

Under similar assumptions, the same is true for the random measurment noise statistics:

$$\sigma_{\nu_{\theta_x}} = \sigma_{\nu_{\theta_y}} = N/\sqrt{2} \qquad \sigma_{\nu_R} = K\sqrt{2}N \tag{13}$$

where N is the standard deviation of the four-head model random measurement error. [In Eqs. (12) and (13), K is a constant which transforms the error in α expressed in arc seconds into a range error expressed in feet. For this 5× orbit, the value of K is approximately 5.54×10^4 ft/arc·sec.)

Error Source Summary

In general, a navigation filter analysis investigates the filter's performance in the presence of three types of errors—modeling errors, measurement errors, and initial state estimation errors. For this study there are no modeling errors, since the optimal covariance analysis assumes that the filter models the satellite dynamics exactly. The measurement errors for the star sensor are specified by the standard deviations, σ_β and σ_λ, of the random errors in Eq. (6). Usually there are assumed equal and may be referred to as the star sensor measurement accuracy, σ_{SS} ($\sigma_\beta = \sigma_\lambda - \sigma_{SS}$). The measurement errors for the Earth sensor are specified by the standard deviation, $\sigma_{ES} = N$, of the random noise of the equivalent four-head horizon sensor, as in Eqs. (10) and (13). For the range measurement, the value of σ_{ν_R} of Eq. (13) may be specified.

There are a maximum of 15 initial state estimation errors, which are the standard deviations (1σ) of the errors in the filter's estimats of vehicle position and velocity (three components each), vehicle alignment with respect to an inertial reference (three components), star sensor alignment bias with respect to the vehicle (three components), and the Earth sensor measurement biases (three components). These estimation errors are the square roots of the diagonal elements of the initial estimation error covariance matrix of Eq. (2).

Navigation Filter Performance Results

General Filter Behavior

Figure 5 shows the 1σ estimation error time histories of 11 of the 15 filter states for a typical simulation of the navigation system. (Only the z components of alignment and star sensor bias errors are shown. The x and y components of each are very similar to the z component.) The star sensor and Earth sensor measurement errors are 2 and 50 arc·sec, respectively; with measurements taken every 5 min. The initial and final (steady-state) error statistics are tabulated in Table 1 (set No. 1). The navigation begins at $t_0 = 12$ h and continues for a period of 15 days. In this time, the spacecraft undergoes an eccentric anomaly change of approximately 484 deg.

The error histories show that the convergence time for the position and velocity estimates is approxiamtely 60 h or 2.5 days. In this time the spacecrft covers an eccentric anomaly angle of approximately 80 deg. This slow rate of convergence is due to the slow movement of the spacecraft in its orbit. The filter attempts to estimate position and velocity from the Earth sensor (ES) measurements of the direction to the Earth's center. This direction changes slowly due to the low orbital velocity, so tht new measurement information is supplied at a low rate, resulting in a long convergence time for the position error estimates. Since no velocity information is included in the measurements, velocity is derived by differencing position information. Thus the slow position estimate convergence causes slow velocity estimate convergence.

Table 1 Initial and final estimation errors

Filter state	Set No. 1 initial error	Set No. 1 error after 15 days	Set No. 2 initial error
Position (ECI)			
x, ft	3.0×10^4	...	3.0×10^4
y, ft	1.5×10^4	...	1.5×10^4
z, ft	1.5×10^4	...	1.5×10^4
rss, n.mi.	6.0	2.72	6.0
Velocity (ECI)			
$\dot{x}$, ft/s	5.0	...	0.2
$\dot{y}$, ft/s	5.0	...	0.2
$\dot{z}$, ft/s	5.0	...	0.2
rss, ft/s	8.7	0.115	0.35
Alignment			
ϕ_x, arc-sec	2.0	...	40.0
ϕ_y, arc-sec	2.0	...	40.0
ϕ_z, arc-sec	2.0	...	40.0
rss, arc·sec	3.5	2.412	69.3
Star sensor alignment bias			
ϕ_{SS_x}, arc-sec	2.0	1.360	5.0
ϕ_{SS_y}, arc-sec	2.0	1.403	5.0
ϕ_{SS_z}, arc-sec	2.0	1.414	5.0
Earth sensor bias[a]			
θ_x, arc-sec	10.0	0.560	35.36
(B)	(14.14)		(500.)
θ_y, arc-sec	10.0	4.463	35.36
R, ft	8.6×10^5	46,170	3.04×10^6
(B)	(15)		(50.0)

[a] Equivalent four-head sensor standard deviations given in parentheses.

The large peak in the position error early in the trajectory is due to the velocity error, the integral of which results in position error. As the velocity error is decreased, the contribution of its integral to position error is reduced. The position error peak is greatly reduced by a better initial velocity estimate.

The alignment errors and the star sensor bias errors converge to their steady-state values upon processing of the first measurement. The star sensor (SS) measurement is the source of the alginment information, so that the residual bias in the star sensor perpetuates the error in the alignment estimate. The alginment error does not decrease significantly after the first measurement, since the vehicle is assumed to be in an attitude hold configuration and subsequent SS measurements repeatedly measure the same quantity, yielding no new information.

The estimation errors of the three Earth sensor biases converge at approximately the same rate as the position and velocity errors. As the vehicle moves about its orbit, the orientation of the vector to the Earth's center is changing, supplying new information to the filter. As a result, the filter is able to calibrate the ES bias. The process is slow, however, due to the slow movement of the vehicle.

If the measurement frequency is increased, somewhat smaller position errors can be achieved (approximately 2000 ft rss for the case shown), while other errors remain the same. However, one would not expect filter accuracy to improve without limit as more frequent measurements are made. If measurements are made too frequently, the vehicle state does not change appreciably between measurements (due to its low orbital velocity), and no new information is derived from the added measurements, so estiamtion errors are not decreased.

Removal of the Earth sensor range measurement has no effect on filter performance other than the fact that the range bias error does not decrease from its original value. This is because the intial position estimation errors are small relative to the range measurement error, so that the filter deweights the position information acquired from the range measurement in favor of the propagated position information. If the initial position errors were significantly larger or the range measurement errors small, it is expected that the Earth sensor range measurement would play a more important role in supplying position information.

Filter Sensitivity to Initial Estimation Errors

The determination of the variables which most influence the filter's accuracy has been accomplished by conducting a seris of simulations in which the initial estimation errors were varied systematically. This simulations show that the initial velocity estimation error has little effect on the steady-state filter performance. However, a smaller initial error results in a smaller transient peak in each of the position component estimation errors. Position and velocity errors are also relatively independent of initial Earth sensor bias estimates.

Degradations in initial alignment errors have only a small influence on steady-state position and velocity accuracy, provided the star sensor bias errors are small. Similarly, poorer initial estimates of star sensor biases degrade filter accuracy only slightly, provided there is good initial alignment. However, if the star sensor bias degradation is coupled with a degradation in initial alignment accuracy, the navigation accuracy is seriously degraded. This is illustrated in Table 2, which tabulates steady-state rss position and velocity errors (after 15 days of navigation) as functions of initial star sensor (SS) bias error, initial alignment error, and initial position error. (The initial velocity estimation error is 0.2 ft/s each direction, and the Earth sensor bias error is 50 arc·sec. The star and Earth sensor measurement errors are 2 and 50 arc·sec, 1σ, respectively.) Assuming a 6-n.mi. initial position error (above the diagonal line in each table entry), for a small initial alignment error, the navigation accuracy is not degraded as the initial SS bias is increased (moving down

Table 2 Steady-state rss position and velocity errors as functions of initial alignment, star sensor bias, and position estimation errors

Initial SS bias error, σ	Initial alignment error, σ: 2 arc-sec (above diagonal)	2 arc-sec (below diagonal)	40 arc·sec (above diagonal)	40 arc·sec (below diagonal)
2 arc·sec	$P=16{,}370$ ft[a] $V=0.1170$ ft/s	$V=0.1903$	$P=17{,}330$ ft $V=0.1206$ ft/s	$P=28{,}800$ $V=0.1932$
5 arc·sec	$P=17{,}170$ ft $V=0.1197$ ft/s	$P=28{,}680$ $V=0.1925$	$P=21{,}000$ ft $V=0.1422$ ft/s	$P=32{,}310$ $V=0.2168$
20 arc·sec	$P=17{,}320$ ft $V=0.1206$ ft/s	$P=28{,}790$ $V=0.1932$	$P=33{,}510$ ft $V=0.2109$ ft/s	$P=51{,}740$ $V=0.3573$

[a] Above diagonal initial position error is ~6 n. mi. rss.
[b] Below diagonal initial position error is ~60 n. mi. rss

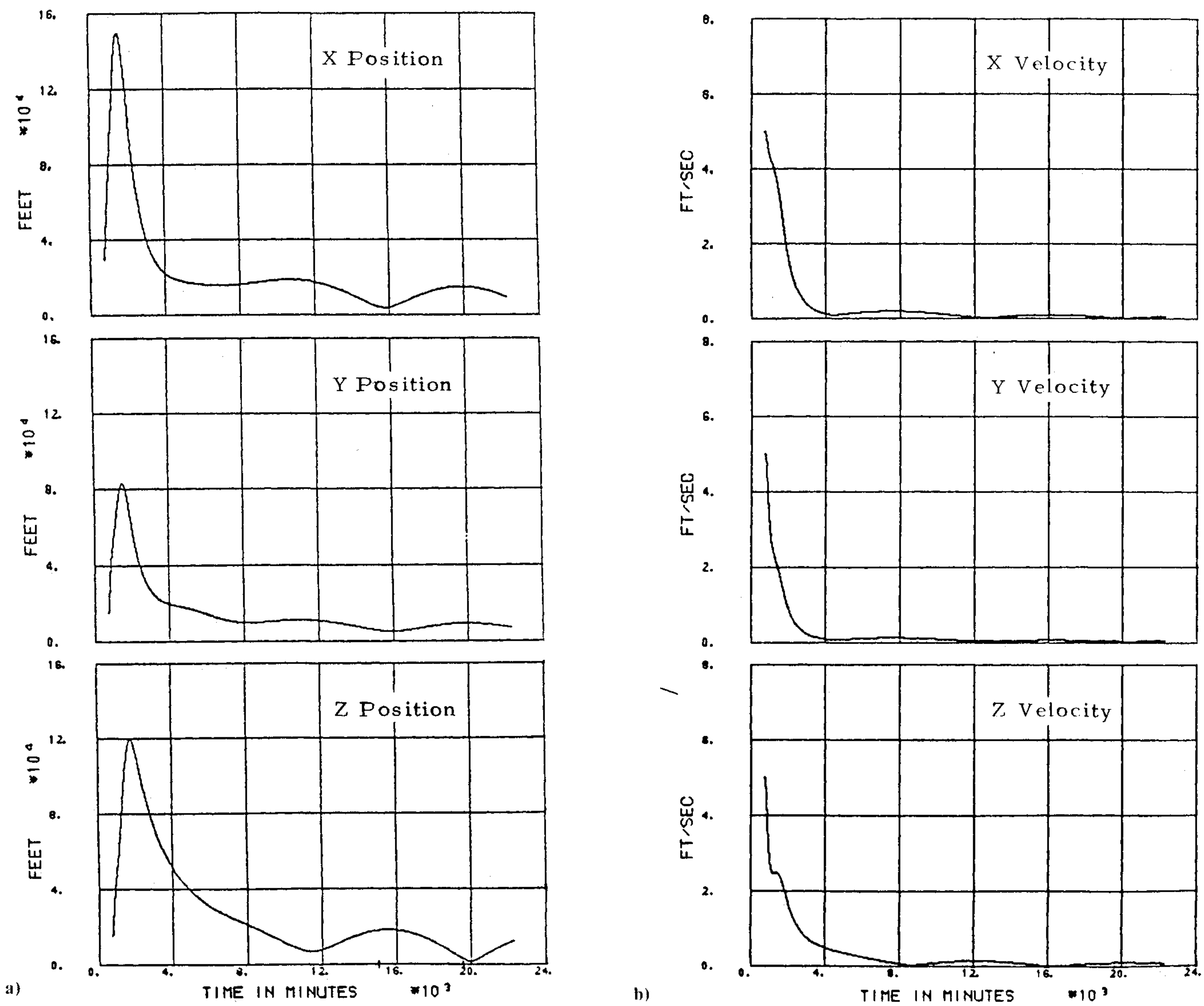

Fig. 5 Estimation error propagation (P set No. 1; $\sigma_{ES}=50$ arc·sec; $\sigma_{SS}=2$ arc·sec; $\Delta t=5$ min).

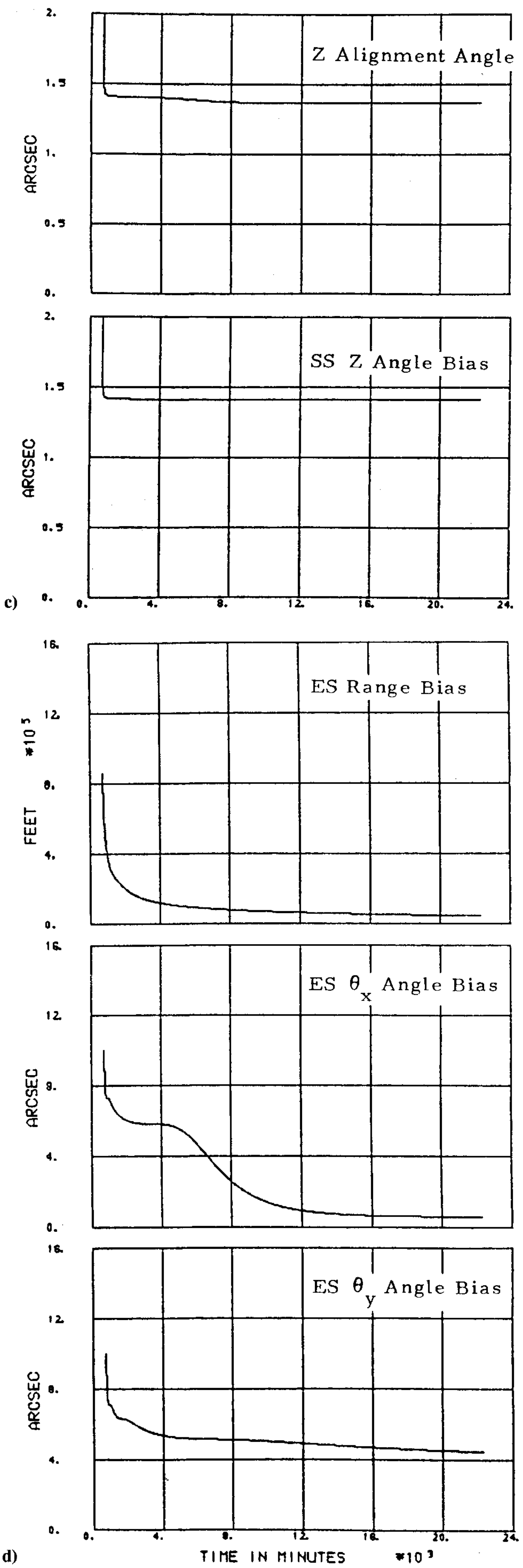

Fig. 5 (continued)

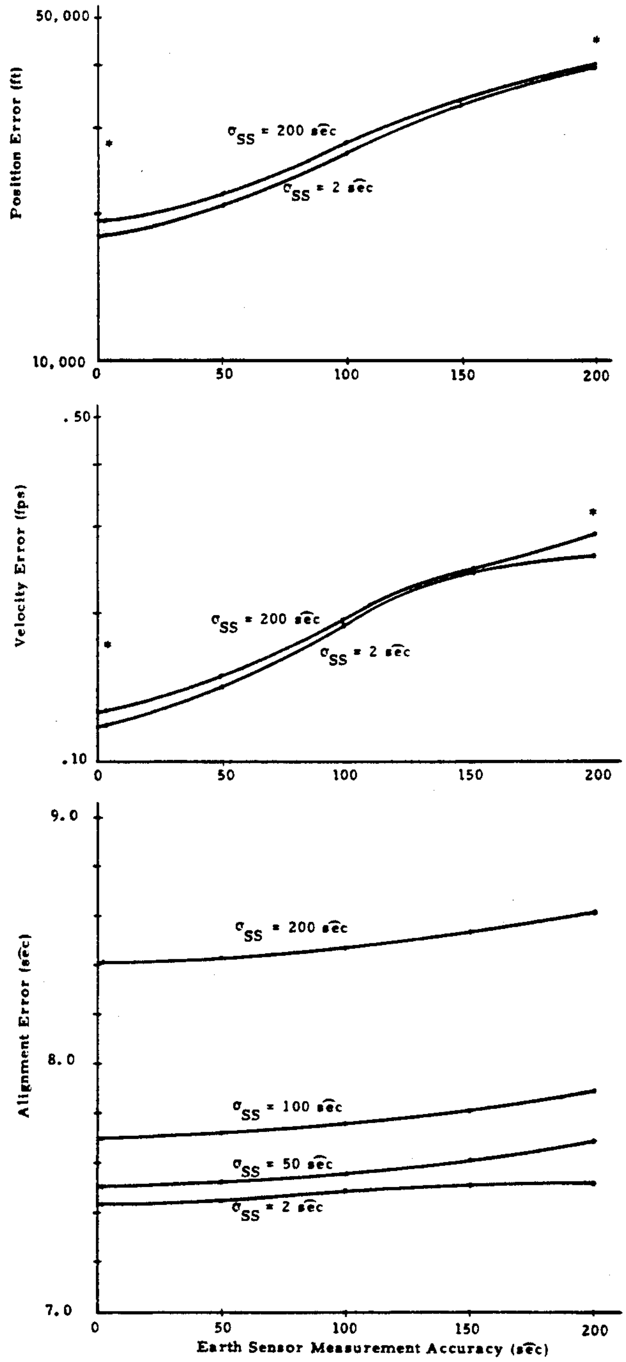

Fig. 6 Steady-state rss position, velocity, and alignment acccuracy as a function of Earth sensor and star sensor measurement accuracy.

firs column), so much as is the case when the initial alignment error is larger (second column). Conversely, a larger SS bias results in relatively more navigation error as the initial alignment error increases (moving across a row). In addition, for a given combination of alignment and SS bias errors, the steady-state position and velocity errors are increased if the initial position error is increased (below diagonal line vs above diagonal line).

For a given initial position error, the navigation errors are largest when both initial alignment estimates and initial SS bias estimates are poor. With larger SS biases, the error in the transformation of the sensor measurements into the filter coordinate system [Eq. (7)] is larger. Therefore, even though there is little noise (accurate SS measurements), the measurements are not able to provide sufficiently accurate alignment information to the filter to reduce large initial alignment estimation errors. At least one source of accurate alignment information is necessary.

Filter Sensitivity to Measurement Errors

The other classification of error sources, in addition to initial estimation errors, is the measurement error. It is of interest to determine the naviation performance subject to different magnitudes of star sensor and earth sensor measurement noise. This gives an indication of the instrument quality which is needed to meet a given mission requirement on naviation accuracy.

Figure 6 plots the navigation errors after 15 days as a function of star sensor and Earth sensor measurement noise. The initial estimation errors are those of set No. 2 in Table 1, which are believed to be realistic errors. The measurement set consists of the star sensor angles and he Earth sensor angles (no range) taken every 5 min. An unexpected result is revealed. For the initial estimation errors assumed, the steady-state navigation accuracy depends very little on the accuracy of the star sensor measurements. In fact, even when the star sensor measurements are eliminated (indicated by the asterisks at $\sigma_{ES} = 2$ and 200 arc·sec in Fig. 6), neither the position nor the velocity errors increase appreciably. An examination of the error histories for the case with star sensor and Earth sensor accuracies of 200 arc·sec shows that the estimation errors conveerge more slowly than is the case with small emasruement errors. But the estimation errors do converge. The conclusion can therefore be made that the filter containing an exact model of the actual process dynamics (the optimal filter) is able to maintain, and slightly improve upon, intial estimation errors of the magnitude of those in set no. 2 of Table 1, utilizinging a star sensor and an Earth sensor of quite ordinary accuracies.

Conclusion

This study investigates the performance of a navigation system operating autonomously onboard a satellite at an altitude of 5× synchronous. A star sensor and Earth sensor provide measurements to augment the position, velocity, and attitude stare estimates maintained by the filter.

The study results provide insight into the quantitative and qualitative characteristics of the 5× autonomous navigation system, including its sensitivities and the significant variable interactions. The general filter behavior shows a convergence time of approximately 2½ days or 80 deg of eccentric anomaly. The slow convergence is due to the vehicle's slow change in position. The filter must process measurments made at widely different positions before it can converge. The position error transient shows a peak, the magnitude of which is dependent on the initial velocity error. Experiments with measurement frequency show that navigation errors decease slightly if measurements are taken more frequently. If the initial position estimation errors are small relative to the Earth sensor range measurement accuracy, the range measurement contributes little to filter accuracy other than aiding in the estimation of the range bias.

The study of filter sensitivity to initial estimation errors shows that initial position error and the combination of poor initial alignment and large star sensor bias errors are detrimental to navigation accuracy. However, if only the initial alignment is poor or only the star sensor bias errors are large, the filter performs well. Only one source of accurate alignment information (accurate initial alignment or an accurate star sensor) is necessary. An increase in initial velocity error results in a larger position transient peak, but has little steady-state effect. The initial Earth sensor bias estimates affect the position and velocity estimates only slightly.

An investigation of filter sensitivity to measurement errors shows that, for a set of initial estimation errors believed realistic, the optimal filter is able to maintain its initial accuracy even with star sensor and Earth sensor measurment errors of 200 arc·sec.

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